Integer Flows

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Abstract of a lecture at Keio University, Yokohama

This is an introductory lecture for graduate students who have some basic knowledge in graph theory.

Integer flow was introduced by Tutte as a dual version of the map-coloring problem. This lecture will introduce some basic concepts, fundamental theorems of the area (based on the textbook "Integer flows and cycle covers of graphs" by the instructor), and, some recent progress in this area as well.

1 Notation and definition

Circuit is a connected 2-regular subgraph.

An edge e of G is a bridge of G if e is contained in no circuit of G.

A graph is even if every vertex is of even degree.

A family of even subgraphs is an even subgraph $(1, 2)$-cover if every edge of the graph is covered once or twice by the family.

Definition 1 *Let $G = (V, E)$ be a graph. An ordered pair (D, f) is an integer flow of G if D is an orientation of G and $f : E(G) \mapsto Z$ such that*

$$\sum_{e \in E^-} f(e) = \sum_{e \in E^+} f(e)$$

for every vertex $v \in V(G)$.

An integer flow (D, f) is a k-flow if

$$|f(e)| < k$$

for every edge $e \in E(G)$.

An integer flow (D, f) is nowhere-zero if

$$f(e) \neq 0$$

for every edge $e \in E(G)$.

The support of a flow (D, f) is

$$\{e : f(e) \neq 0\}$$

2 Major conjectures

Theorem 2 *Let G be a planar graph. The G admits a nowhere-zero k flow if and only if G is face-k-colorable.*

Conjecture 3 (5-flow conjecture by Tutte) *Every bridgeless graph admits a nowhere-zero* 5*-flow.*

Conjecture 4 (4-flow conjecture by Tutte) *Every bridgeless graph containing Petersen subdivision admits a nowhere-zero* 4*-flow.*

Conjecture 5 (3-flow conjecture by Tutte) *Every* 3*-edge-connected graph admits a nowhere-zero* 4*-flow.*

Conjecture 6 (Circuit double cover conjecture by Szekeres, Seymour) *Every bridgeless graph has a family of circuits such that every edge is covered twice.*